1.1 Determine for the next system of forces:

a) The magnitude and direction of the force F = (-5,-2, 3) N

b) The moment this force creates about the origin of coordinates when acts at a point A (4, -2, -1) m.

c) Consider now two new vectors b and c: b = (0, -1, 0) and c = (x, -2, 1). Determine the value of *x* for which the three vectors are coplanar.

a) Direction and magnitude

$$F = \sqrt{(-5)^2 + (-2)^2 + 3^2} = 6{,}16\ \mathrm{N}$$

$$\cos\alpha_x = \frac{-5}{6{,}16} = 144{,}19°$$

$$\cos\alpha_y = \frac{-2}{6{,}16} = 71{,}05°$$

$$\cos\alpha_z = \frac{3}{6{,}16} = 60{,}85°$$

Then the vector is completely characterized by it modulus and its direction.

b) Moment about point A

In this exercise in order to determine the moment we will calculate the cross product through a determinant.

$$M_O^F = \begin{vmatrix} i & j & k \\ 2 & 1 & 2 \\ 1 & -4 & 0 \end{vmatrix} = 2j - 8k - k + 8i$$

$$\mathbf{M_O^F = (8\,, 2\,, -9)\ N.m}$$

c) Value of x for which the three vectors are coplanar.

The necessary and sufficient condition for three vectors to be coplanar is that its mixed triple product shall be equal to zero.

We understand mixed triple product as is defined as the dot product of one of the vectors with the cross product of the other two. This is:

$$\mathbf{a.(bxc) = 0}$$

Now for the three vectors given in the statement:

$$a.(bxc) = \begin{vmatrix} 6 & -2 & 3 \\ 0 & -1 & 0 \\ x & -2 & 1 \end{vmatrix} = -6 + 3x = 0$$

Thus, the necessary condition is these vectors to be coplanar are simply given by the value of x:

$$\boxed{\mathbf{x = 3}}$$

1.2 Answer the next questions:

a) Let's consider a couple of reference systems Oxyz and O′ x′ y′ z′. What is the relation of the moment created by a force F when it is calculated about O and O′. What conclusion can be applied to a couple system of forces?

b) Is it possible that the cross product of two vectors $\vec{a}$ and $\vec{b}$ would be equal to zero even though none of them is equal to zero?

c) Express the equilibrium conditions in function of the linear and angular moment

a) Moment

The moment of the force with respect O is:

$$\mathbf{M_O^F = r\ x\ F}$$

Now considering that: $r' = 0'0 + r$

$$M_{O'}^F = r' x\,F = (0'0 + r) x\,F =$$

$$0'0\,x\,F + r\,x\,F = M_O^S + 0'0\,x\,F$$

Using this expression for a couple system leads to:

$$\mathbf{M^S_{O'}} = \mathbf{M^S_O}$$

Therefore, the resultant moment of a couple system remains constant, independently of the reference point because a couple system has a resultant force equal to zero.

b) Cross product

$a \times b = a.b.\sin\theta$

Then if a≠ 0 and b is ≠ 0 the only way to be zero is that $\sin\theta = 0 \rightarrow \theta = {}^{\pi}/_{2} + n\pi$ so it happens when they are perpendicular

c) Equilibrium

Considering the second Newton law:

$$\sum F = ma$$

Thus, if there is no acceleration, there is a null force with remains constant until acted upon a force. Now for the exercise:

Considering the linear momentum:

$P = mv$

$\frac{dP}{dt} = m\frac{dv}{dt} = ma \rightarrow a = 0 \rightarrow P = cte$

If the linear momentum is constant, it means acceleration is also equal to zero and then the system will be under static equilibrium.

$L = r \times mv$

$\frac{dL}{dt} = \frac{dr}{dt} \times mv + r \times m\frac{dv}{dt}$

$\frac{dr}{dt} = v \rightarrow \frac{dr}{dt} \times mv = 0\ (\alpha = 0)$

$r \times m\frac{dv}{dt} = r \times \frac{dP}{dt}$

$\frac{dP}{dt} = m\frac{dv}{dt} = ma \rightarrow a = 0 \rightarrow L = cte$

This result means that if the linear momentum or the angular momentums are equal to zero, then it means that the system is in equilibrium.

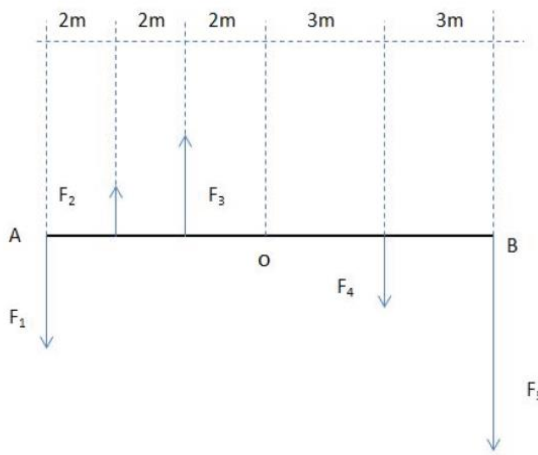
1.3 Calculate the resultant moment of the forces shown in the figure with respect point O, knowing the value of the system of forces of the figure.

$\mathbf{F_1 = 100\ N}$

$\mathbf{F_2 = 30\ N}$

$\mathbf{F_3 = 80\ N}$

$\mathbf{F_4 = 120\ N}$



The first step is to define the sign criteria. For example, in this case we will consider that forces that creates a clockwise moment about point O. In case of counterclockwise sense, the result would be equal in modulus but opposite in sign, indicating which is really its correct direction.

$M_o = -6.100 + 30.4 + 80.2 + 3.20 + 120.6$

$M_o = 460\ N$

Thus, the moment created by the set of forces tends to rotate the bar AB clockwise.